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# Teaching Conceptual Model–Based Word Problem Story Grammar to Enhance Mathematics Problem Solving

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Borrowing the concept of story grammar from reading comprehension literature, the purpose of this study was to examine the effect of teaching *word problem (WP) story grammar* on arithmetic WP solving that emphasizes the algebraic expression of mathematical relations in conceptual models. Participants were five students in Grades 4 and 5 with or at risk for mathematics disabilities in two urban public elementary schools in the Midwest. An adapted multiple-probe-across-participants design was used to assess the functional relation between the intervention and students' problem-solving performance. The results indicated that conceptual model–based representation prompted by WP story grammar improved students' performance on arithmetic WP solving and promoted prealgebra concept and skill acquisition. The practical implications of integrating algebraic reasoning in elementary mathematics are discussed.

***Keywords:*** *conceptual model–based problem solving; word problem story grammar; mathematics word problem solving; algebra readiness; elementary school mathematics; students with learning disabilities or problems; mathematics disabilities*

According to the National Research Council (Kilpatrick, Swafford, & Findell, 2001), assessments conducted at state, national, and international levels over the past 30 years have indicated that although "U.S. students may not fare badly when asked to perform straight forward computational procedures, they tend to have a limited understanding of basic math concepts," and they are "notably deficient in their ability to apply mathematical skills to solve even simple problems" (p. 4). The mathematics performance of U.S. students may have improved over the years, but the 2003 National Assessment of Educational Progress still reported that 23% of fourth graders and 32% of eighth graders scored below the "basic" level. The problem of underachievement is more severe for students with disabilities. In particular, 50% of fourth grade students with disabilities and about 71% of eighth grade students with disabilities scored below the basic level. These data suggest that many students are still not being given the educational opportunities they need to achieve at high levels (Kilpatrick et al., 2001).

## Promoting Conceptual Model–Based Word Problem (WP) Solving

Problem solving is the cornerstone of school mathematics (National Council of Teachers of Mathematics [NCTM], 2000). Specific problem-solving behaviors distinguish successful problem solvers from poor problem solvers (Mayer, 1999). For instance, successful problem solvers (a) quickly and accurately identify the mathematical structure (e.g., compare) of a problem that is generalizable across a wide range of similar problems, (b) remember a problem's structure for a long time, and (c) distinguish relevant from irrelevant information (Krutetskii, 1976; Quilici & Mayer, 1996). Successful problem solvers look for and find underlying structural information (i.e., problem

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schemata), whereas unsuccessful problem solvers tend to focus on the surface features of problems, which makes it difficult for them to transfer their learning to a wide range of structurally similar problems (Silver & Marshall, 1990). In sum, successful problem solvers build their problem solving on a conceptual model of problem situations (Hegarty, Mayer, & Monk, 1995; Jonassen, 2003).

Historically, many believe that selecting and applying an appropriate arithmetic operation for solution is critical to the success of problem solving. However, according to Jonassen (2003), "contemporary approaches to story problem solving have emphasized the conceptual understanding of the story problems before any solution attempts" (p. 269). Specifically, a *conceptual model* that recognizes and reorganizes the deep structure of a problem (i.e., the problem schema) needs to be constructed before solution planning. Furthermore, a conceptual model should drive the development of a solution plan that involves selecting and applying appropriate arithmetic operations. Because problems with the same problem schema share a common underlying structure requiring similar solutions (Chen, 1999; Gick & Holyoak, 1983), it is suggested that students need to learn to understand the structure of the mathematical relationships in WPs and that students should exhibit this understanding through creating and working with meaningful representation (Brenner et al., 1997).

## Teaching Conceptual Model–Based WP Story Grammar to Facilitate Meaningful Representation

In the early 1900s, anthropologists found that people follow a pattern when retelling stories they have read or heard, regardless of age or culture. This pattern is referred to as *story grammar* (Dimino, Gersten, Carnine, & Blake, 1990; Mandler & Johnson, 1977; Stein & Glenn, 1979). In story grammar, *grammar* means "elements." Therefore, story grammar addresses the elements of a story. Story grammar involves a set of expectations or knowledge about the internal structures of stories (which can be conceptualized as story schemata; Rand, 1984) that makes both comprehension and recall more efficient. Story grammar instruction directs attention to key elements of stories and provides students with a specific structure for the organization of text structure (i.e., "story mapping"; Boulineau, Fore, Hagan-Burke, & Burke, 2004, p. 106). Story grammar aims to improve students' reading comprehension by giving them a framework they can use when reading stories (e.g., by asking a series of story grammar questions regarding who, what, where, when, and why). Consistent use of the same questions about stories equips students with a framework they can apply on their own (Gurney, Gersten, Dimino, & Carnine, 2001). Research demonstrates that explicit instruction in both story grammar and *story mapping* has positive effects on the reading comprehension skills of elementary and secondary students with and without learning disabilities (e.g., Boulineau et al., 2004; Dimino et al., 1990, Gardill & Jitendra, 1999). Story grammar and story mapping can serve as tools to assist students in organizing and representing the internal structures of stories and therefore improve their comprehension (Sorrell, 1990).

By definition, "a text structure that is common to most narrative stories is story grammar" (Gardill & Jitendra, 1999, p. 2). Similarly, a WP story structure that is common across a group of WP situations can be defined as *WP story grammar* for a particular problem type. Borrowing the concept of story grammar from reading comprehension literature, the first author of this study designed a set of WP story grammar self-questioning prompts that emphasize the algebraic expression of mathematical relations in WP conceptual models to assist meaningful representation and problem solving. Generally speaking, *part-part-whole* (PPW; part + part = whole) is a generalizable conceptual model in addition and subtraction WPs in which part, part, and whole are the three basic elements. In contrast, *factor-factor-product* (factor × factor = product) is a generalizable conceptual model in multiplication and division arithmetic WPs in which factor, factor, and product are the three basic elements. It should be noted that the three basic elements (in either the PPW or factor-factor-product model) will have unique denotations when specific problem subtypes apply. For example, in a *combine* problem type (e.g., "Emily has 4 pencils and Pat has 8 pencils. How many pencils do they have all together?"), the number of pencils Emily has and number of pencils Pat has are the two parts; these two parts make up the combined amount (i.e., "all together"), or the whole. In contrast, in a *change* problem type (e.g., "Susan had 12 candies. She gave 4 to Tom. How many candies does Susan have now?"), the number of candies Susan had in the beginning is the whole amount, whereas the number of candies Susan gave away and the number of candies she has now are the two parts that make up the whole or the beginning amount.

Because WPs of a specific problem type (e.g., PPW) share a common underlying structure involving the same key elements (e.g., part, part, and whole), a set of WP story grammar questions can be generated to serve as prompts in guiding students when they organize information and express mathematical relations in WP conceptual models. For instance, in the PPW problem types, basic WP story grammar questions such as "Which sentence tells about the whole or combined quantity?" and "Which sentence tells about one of the small parts that makes up the whole?" can assist in the comprehension and representation of the underlying structure of a WP in the conceptual model (i.e., part + part = whole), therefore facilitating solution planning. Emphasis on the meaningful representation of mathematical relations in problem solving is consistent with contemporary approaches to story problem solving that emphasizes conceptual understanding of the story problems before decision making on the *choice of operation*. In addition, an emphasis on representing mathematical relations in conceptual models facilitates algebraic reasoning and thinking that involves symbolic expressions of mathematical relations in equations (Kilpatrick et al., 2001; Moses, 1997). This is consistent with the National Research Council's call that "the basic ideas of algebra as generalized arithmetic" (Kilpatrick et al., 2001, p. 419) and "algebraic ways of thinking" should be introduced "well before" students get to "the formal study of algebra" (p. 13).

In the past decade or so, schema-based instruction has shown potential benefits in teaching mathematics problem solving to students with and without disabilities. Specifically, researchers have investigated the effect of schema-based instruction on teaching algebra and arithmetic WP solving (e.g., Hutchinson, 1993; Jitendra et al., 1998; Xin, Jitendra, & Deatline-Buchman, 2005; Zawaiza & Gerber, 1993) and transferring learned problem solution rules to novel problems (e.g., Fuchs, Fuchs, Finelli, Courey, & Hamlett, 2004; Fuchs et al., 2003). However, existing schema-based instruction in teaching arithmetic WP solving to elementary students with learning disabilities is primarily *arithmetic approach* oriented, emphasizing either concrete representation (e.g., the use symbols to represent or lay out each object or item in a set) or decision making on the choice of operation (e.g., "When the total is the unknown, you add"; "When the compared is the unknown and is part of the referent amount, you divide"). The former may not be efficient or feasible when the numbers in a problem get large. The latter relies on decision making on the choice of operation that is confined to specific WP situations. Because the placement of the unknown in a problem can alter the choice of operation from addition to subtraction or multiplication to division, students have to remember or rely on rules for all types and apply the correct rule to figure out the operation for a solution. Unlike a rule-driven or arithmetic-oriented approach, a generalizable conceptual model–based problem-solving approach involves *symbolic* or *algebraic expressions of mathematical relations in equations* in which one or more of the quantities are unknown and finding the unknown value in the equation. Students do not have to remember numerous rules for different problems. This approach may particularly benefit students with learning disabilities because of their cognitive disadvantage in attention, organization, and working memory (Gonzales & Espinel, 1999, Zentall & Ferkis, 1993).

The purpose of this study was twofold. First, it was designed to study the effect of teaching elementary students with mathematics disabilities or difficulties (MD) and those at risk for MD conceptual model–based WP story grammar on arithmetic WP solving. Second, because the conceptual model–based WP story grammar designed for this study emphasized the algebraic expression of mathematical relations in problem representation and solution, another purpose of this study was to assess the effect of conceptual model–based problem solving (i.e., the intervention) on improving students' prealgebra concept and skills. On the basis of these two purposes, two research questions were addressed: (a) Is there a functional relationship between the intervention and students' improved WP-solving performance? and (b) Does the intervention improve students' prealgebra concept and skills?

## Method

### Participants and Setting

Participants were 5 fourth and fifth grade students with or at risk for MD. These students were enrolled in after-school programs at two small, urban public elementary schools in the Midwest. Students identified by classroom teachers as being at risk for academic failure were eligible for the after-school programs. Because there were numerous enrichment programs, such as computer-assisted programs for self-paced learning in various subject areas and science and social studies learning projects offered to the after-school program students, only 10 students (4 fourth graders and 6 sixth graders) consented to participate in the

**Table 1**
**Student Demographics**

| Variable | Deb[a] | Rita | Kate | Matt | Bart |
|---|---|---|---|---|---|
| Gender | Female | Female | Female | Male | Male |
| Ethnicity | Caucasian | Caucasian | Caucasian | Caucasian | Caucasian |
| Age | 11 years | 10 years 3 months | 11 years | 11 years | 10 years 5 months |
| Grade | 5 | 4 | 4 | 4 | 4 |
| Classification | NL | LD | LD | NL | NL |
| Socioeconomic status[b] | Low | Low | Low | Low | Medium |
| Reduced-price/free lunch | Free | Reduced price | Free | No | No |
| Years in special education | 0 | 1 | 3 | 0 | NA |
| Learning support classroom placement | NA | Reading/math | Language arts | NA | Reading/math |
| % in general education class | 100% | 60% | 40% | 100% | 80% |
| IQ (percentile) | WISC-III | WISC-IV | WISC-IV | OLSAT | NA |
| Full scale | 23 | 23 | 9 | 16 | |
| Verbal | 42 | 19 | 5 | 10 | |
| Performance | 12 | 55 | | | |
| Working memory | | | 6 | 25 | |
| Perceptual reasoning | 8 | | 14 | | |
| Achievement (percentile) | WJTA-III | WIAT-II | BASIS | NA | MAP |
| Math composite | 40 | | 19 | | Unsatisfactory |
| Math reasoning | 32 | 10 | | | Low |
| Numerical operations | | 23 | | | Low |
| Composite reading | 45 | 23 | 4 | | Low |
| Composite writing | 35 | 3 | | | Low |
| KeyMath-R/NU (percentile) problem solving | 17 | 20 | 7 | 40 | 31 |

Note: NL = not labeled; LD = learning disability (determined by the case conference committee on finding that a severe discrepancy existed between a student's academic achievement and normal or near normal potential); NA = not applicable or not available; WISC-III = *Wechsler Intelligence Scale for Children–III* (Wechsler, 1991); WISC-IV = *Wechsler Intelligence Scale for Children–IV* (Wechsler, 2003); OLSAT = *Otis-Lennon School Ability Test* (Otis & Lennon, 1995); WJTA-III = *Woodcock-Johnson III Tests of Achievement* (Woodcock, McGrew, & Mather, 2001); WIAT-II = *Wechsler Individual Achievement Test–II* (Psychological Corporation, 2001); BASIS = *Basic Achievement Skills Individual Screener* (Psychological Corporation, 1983); MAP = *Measure of Academic Progress* (The Northwest Evaluation Association, n.d.); KeyMath-R/NU = *KeyMath-Revised/Normative Update* (Connolly, 1998).
a. Deb's scores on the WISC-III and the WJTA-III were taken when she was in second grade. Recently, she was reported to have failing grades in mathematics in classroom assessment and high-stakes testing.
b. Based on parents' professions.

study. Among them, 3 fifth graders did not meet the selection criterion, and 2 fifth graders withdrew from the program during the preassessment because of numerous testing. In this study, MD were determined by a *KeyMath-Revised/Normative Update* (KeyMath-R/NU; Connolly, 1998) Problem Solving subtest score less than the 30th percentile (Geary, 2004). Students at risk for MD were determined by a KeyMath-R/NU Problem Solving subtest score between the 30th and 40th percentiles, inclusive. We chose this selection criterion because it includes those students with specific deficits in arithmetic problem solving rather than more general learning disabilities (Geary, 2004). Table 1 presents demographic information on the 5 participants.

All instruction and testing were conducted in vacant teacher conference rooms or classrooms. It should be noted that during the intervention of this study, the participating students were learning fractions and probability (fourth graders) and operations with fractions (fifth graders) in their regularly scheduled math classes during the school day.

## Dependent Measures

*Criterion WP-solving tests*. The first author designed sets of alternate addition and subtraction as well as multiplication and division WP-solving criterion tests for use at baseline, during the intervention, and for maintenance assessment. These criterion tests were designed in alignment with NCTM (2000) standards, which emphasize varying construction of WPs for assessing conceptual understanding of mathematics problem solving (Cawley & Parmar, 2003).

**Table 2a**
**Sample Problems in Criterion Tests: Addition and Subtraction**

| Problem Type | Sample Problem Situation |
|---|---|
| **Part-Part-Whole** | |
| | **Combine** |
| Part (or smaller group) unknown | Jamie and Daniella have found out that together they have 92 books. Jamie says that he has 57 books. How many books does Daniella have? |
| Whole (or larger group) unknown | Victor has 51 rocks in his rock collection. His friend, Maria, has 63 rocks in her collection. How many rocks do the two have altogether? |
| | **Change-Join** |
| Part (or smaller group) unknown | Luis had 73 candy bars. Then, another student, Lucas, gave him some more candy bars. Now he has 122 candy bars. How many candy bars did Lucas give Luis? |
| | A girl named Selina had several comic books. Then, her brother Andy gave her 40 more comic books. Now Selina has 67 comic books. How many comic books did Selina have in the beginning? |
| Whole (or larger group) unknown | A basketball player ran 17 laps around the court before practice. The coach told her to run 24 more at the end of practice. How many laps did the basketball player run in total that day? |
| | **Change-Separate** |
| Part (or smaller group) unknown | Davis had 62 toy army men. Then, one day he lost 29 of them. How many toy army men does Davis have now? |
| | Ariel had 141 worms in a bucket for her big fishing trip. She used many of them on the first day of her trip. The second day she had only 68 worms left. How many worms did Ariel use on the first day? |
| Whole (or larger group) unknown | Alexandra had many dolls. Then, she gave away 66 of her dolls to her little sister. Now, Alexandra has 63 dolls. How many dolls did Alexandra have in the beginning? |
| **Additive Compare** | |
| | **Compare-More** |
| Larger quantity unknown | Denzel went out one day and bought 54 toy cars. Later, he found out that his friend Gabrielle has 56 more cars than he bought. How many cars does Gabrielle have? |
| Smaller quantity unknown | Tiffany collects bouncy balls. As of today she has 93 of them. Tiffany has 53 more balls than her friend, Elise. How many balls does Elise have? |
| Difference unknown | Logan has 117 rocks in his rock collection. Another student, Emanuel, has 74 rocks in his collection. How many more rocks does Logan have than Emanuel? |
| | **Compare-Less** |
| Larger quantity unknown | Ellen ran 62 miles in one month. Ellen ran 29 fewer miles than her friend Cooper. How many miles did Cooper run? |
| Smaller quantity unknown | Kelsie said she had 82 apples. If Lee had 32 fewer apples than Kelsie, how many apples did Lee have? |
| Difference unknown | Deanna has 66 tiny fish in her aquarium. Her dad Gerald has 104 tiny fish in his aquarium. How many fewer fish does Deanna have than Gerald? |

The addition and subtraction WP-solving criterion tests comprised 14 one-step arithmetic WPs involving two distinguishable problem types, PPW and *additive compare* (AC), that included five subtypes: combine (with two variations), change-join (with three variations), change-separate (with three variations), AC-more (with three variations), and AC-less (with three variations) (Van de Walle, 2004). Multiplication and division WP-solving criterion tests entailed 12 one-step WPs involving two distinguishable problem types, *equal group* (EG) and *multiplicative compare* (MC), each with three variations (Van de Walle, 2004). Tables 2a and 2b present sample problems to illustrate each variation. As shown in these tables, the construction of each WP item was systematically varied in reference to the structure of specific problem schemata and the unknown's position in a problem so that a range of WPs involving four basic operations were represented.

Alternate forms of criterion tests were equivalent in problem construction with reference to types. The second author created a computer program to aid in the generation of multiple equivalent test forms. The computer program ensured the proper distribution of problem subtypes on each test form. Problems generated for each subtype shared a common problem structure that varied only by names used and objects discussed.

**Table 2b**
**Sample Problems in Probes: Multiplication and Division**

| Problem Type | Sample Problem Situation |
|---|---|
| **Equal Group** | |
| Unit rate unknown | A school arranged a visit to the museum in Lafayette Town. It spent a total of $667 buying 23 tickets. How much does each ticket cost? |
| Number of units (sets) unknown | There are a total of 575 students in Centennial Elementary School. If one classroom can hold 25 students, how many classrooms does the school need? |
| Product unknown | Emily has a stamp collection book with a total of 27 pages, and each page can hold 13 stamps. If Emily filled up this collection book, how many stamps would she have? |
| **Multiplicative Compare** | |
| Compared set unknown | Cameron has a brother named Isaac. Cameron has 22 times as many marbles as Isaac. Isaac has 11 marbles. How many marbles does Cameron have? |
| Referent set unknown | Gina has sent out 462 packages in the last week for the post office. Gina has sent out 21 times as many packages as her friend Dane. How many packages has Dane sent out? |
| Multiplier unknown | It rained 147 inches in New York one year. In Washington D.C., it only rained 21 inches during he same year. The amount of rain in New York is how many times the amount of rain in Washington D.C.? |

The names used were randomly selected from a list of the 500 most common male names and the 500 most common female names from the 1990 U.S. census. The objects were selected from a list of common items with which the participants were presumed to be familiar. Numbers used in the WPs were selected first according to operation related to the WP. For addition problems, the addends were chosen according to a discrete uniform distribution on the interval [26, 75], with the exception that the addends were required to be unique. For multiplication problems, the factors were chosen according to a discrete uniform distribution on the interval [11, 30], with the exception that the factors were required to be unique. The intervals were chosen to provide arithmetic operations that could not be readily evaluated by simply guessing but also were not any harder than needed. Computer-generated problems in each probe were reexamined to ensure variation and proper wording in language and situations.

*Standardized measure.* A standardized measure probe was developed to validate the intervention effects on the primary dependent measure (i.e., the criterion WP-solving tests). It included eight problem items taken from the KeyMath-R/NU Problem Solving subtest, four in addition and subtraction and four in multiplication and division problem solving. Only eight items were selected for this measure because these problems represented each one of the four WP types included in this study (i.e., PPW, AC, EG, and MC). Excluded KeyMath-R/NU items entailed number-sense problems (e.g., "What number comes next?"), story making on the basis of pictures presented, and other nonroutine problems, such as finding out missing information in a problem. The KeyMath-R/NU reflects the NCTM standards; its internal consistency reliability is .97 for the entire test and .91 (for fourth graders) and .86 (for fifth graders) for the Problem Solving subtest (Connolly, 1998).

*Prealgebra probes.* Two prealgebra probes were developed to assess the potential improvement of students' prealgebra concept and skills. The *solve equations* probe required students to find the value of an unknown quantity (i.e., the letter *a*) that made an equation true (e.g., $93 = 79 + a$, $196 = a \times 28$). The positions of the unknown were systematically varied across three terms in the equation (i.e., the augend, addend, and sum or the multiplicand, multiplier, and product). Six items were included in either the addition and subtraction probe or the multiplication and division probe. The *algebra expression* probe was designed to test students' prealgebra expression of mathematical relations or ideas. Twelve items (e.g., "Write an expression or equation. Choose a variable for the unknown. Shanti had some stamps. She gave 23 to Penny. Shanti has 71 stamps left") were included in the addition and subtraction probe, and 5 items (e.g., "Antoni has collected 84 autographs. He filled 14 pages in his news autograph album. Each page holds an equal number of autographs. Write an equation with a variable to model this problem") were included in the multiplication and division probe. These items were directly taken from a commercially published mathematics textbook being adopted by the participating schools (Maletsky et al., 2004).

*Scoring*. The percentage of problems solved correctly was used as the dependent measure and calculated as the total points earned divided by the total possible points. Specifically, if the correct answer was given to a problem, one point would be earned.

In addition to scoring correct answers, we also examined whether conceptual model–based representation in the diagram or equation was performed before providing the answer. A graduate student who was naive to the purpose of the study scored all the probes using an answer key. We rescored 40% of the tests. Interrater reliability was computed by dividing the number of agreements by the number of agreements and disagreements and multiplying by 100. Interrater reliability for scoring was 99% (range = 92%–100%) across the independent raters.

## Design

An adapted multiple-probe design (Horner & Baer, 1978) across participants was used to evaluate the functional relationship between the intervention and students' WP-solving performance. A single-subject research design was chosen because the design provides a methodological approach well suited to the investigation of single cases or groups (Kazdin, 1982). In particular, with the multiple-probe design, intervention effects can be demonstrated by introducing the intervention to different baselines or participants at different points in time: "If each baseline changed when the intervention is introduced, the effects can be attributed to the intervention rather than to extraneous events," such as history, maturation, testing, and so on (Kazdin, 1982, p. 126).

On the basis of criterion pretest results, three students were identified as needing intervention in addition and subtraction, and they were taught to solve PPW and AC problems; two students were identified as needing intervention in multiplication and division only (because their pretest performance on addition and subtraction was over 70% correct), and they were taught to solve EG and MC problems. As such, the design included a baseline condition on criterion tests, intervention on the PPW (or EG) problem type, a post-PPW (or post-EG) instruction probe (the probe condition) on the criterion test, intervention on the AC (or MC) problem type, and a posttest (the post condition) following both problem types' instruction. For assessing a generalization effect, students took both the standardized measure and solve equations probe during the baseline and repeated these two measures during the posttest. In addition, all participants took the algebra expression probe following the intervention.

## Procedure

For the cohort receiving instruction in addition and subtraction, all three participants completed one criterion test during the baseline condition. Then, one student (Deb) took two more alternate forms of the criterion test. Following the baseline, the intervention on PPW was first introduced to Deb. Once the data for Deb showed an accelerating trend, the intervention on PPW was introduced to the second participant (Rita) after she took two additional baseline probes immediately before the intervention. The same sequence was followed until all three participants were introduced to the PPW intervention. Following the PPW instruction, probes on the criterion test were administered (as the baseline condition) before AC instruction took place. Posttests were given after both PPW and AC instruction. For the cohort receiving instruction in multiplication and division problem solving, the same procedure was followed.

Participating students received the intervention three times a week, with each session lasting approximately 20 to 35 minutes. Each student received three to six sessions of instruction on PPW or EG, two to three sessions on AC or MC problem instruction, and one to two sessions on solving mixed WPs including both PPW and AC or EG and MC types. Calculators were allowed throughout the study to accommodate participants' skill deficits in calculation.

*Probe condition*. During the probe conditions (i.e., baseline, probe, post, and generalization), participating students were told to read the problems (or call on the examiner if they had difficulty reading) and do their best to solve them. Students were also required to show their complete work. No prompting was provided to students; no feedback was given regarding the accuracy of their solutions or work. Students were provided with sufficient time to complete the tests. No student needed more than 25 minutes to complete a probe. The sequence with which students took alternate forms of the criterion test during baseline and postintervention phases was randomized across participants.

*Intervention procedure*. The first author developed the teaching scripts to guide the strategy teaching throughout the study and to decrease the threat to the integrity of the independent variable. The second author served a primarily role in delivering instruction. Two research assistants administered all probes. In addition, the two assistants were provided with teaching scripts and a checklist that contained critical instructional components to assess the instructor's adherence to the intervention. They observed 40% of the intervention sessions (one to two sessions for each problem type across participants) and judged the presence or absence of each critical component. Treatment fidelity ranged from 88% to 100%, with a mean of 95% across all observed sessions.

Instructions for each of the four problem types (i.e., PPW and AC or EG and MC) were delivered in two parts: *problem-structure* instruction and *problem-solution* instruction. During problem-structure instruction, students learned to identify the problem type or structure and map mathematical relations onto respective conceptual model diagrams (see Figure 1 for conceptual model diagrams for each problem type) using WPs with no unknowns. The purpose of presenting story situations with no unknowns was to provide students with a complete representation of the problem structure of a specific type so that generalized mathematical relations in the problem schema could be visualized.

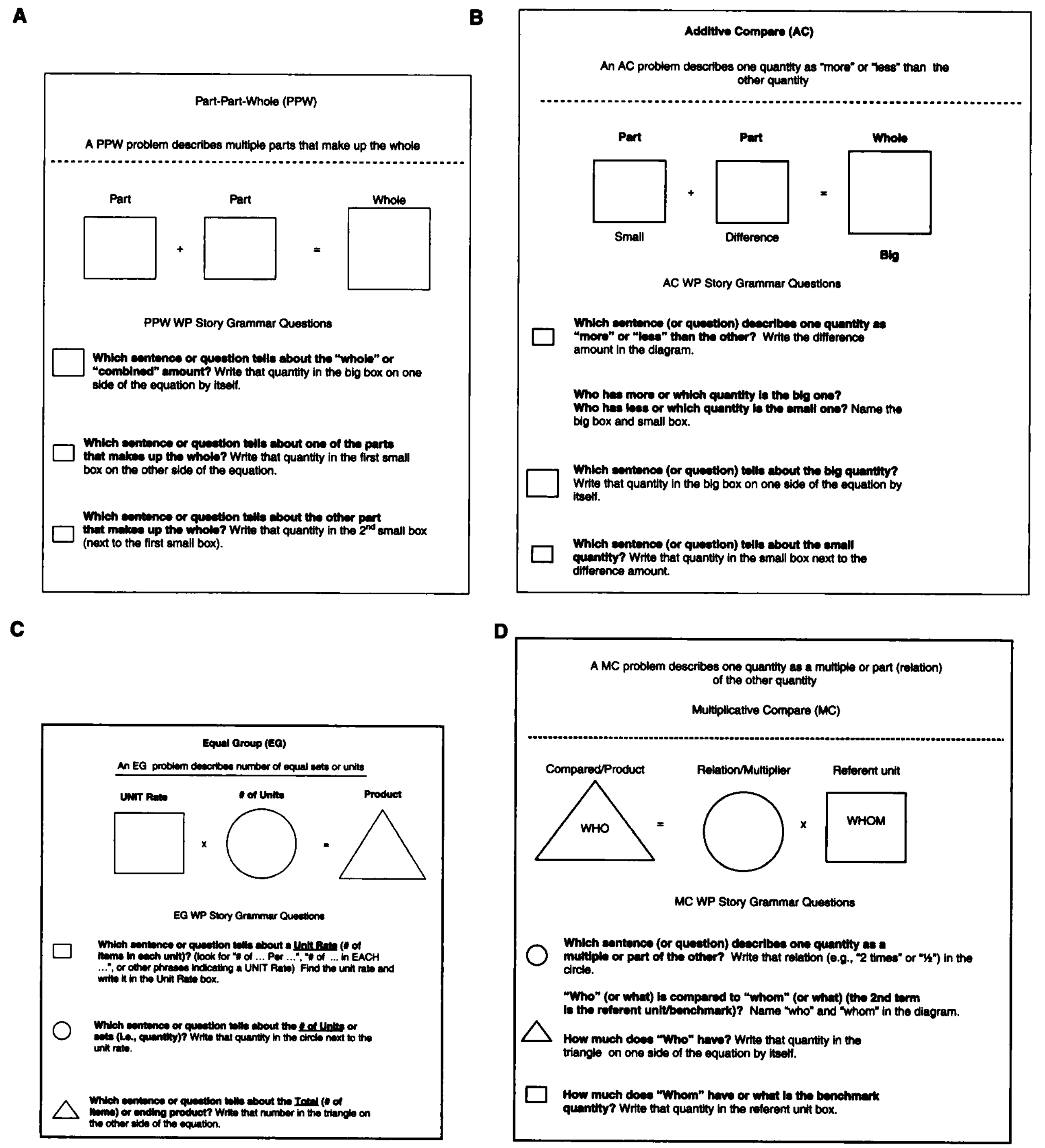
Problem-structure instruction was followed by problem-solution instruction. During problem-solution instruction, WPs with unknown quantities were presented. The first author designed a four-step DOTS (detect, organize, transform, solve) checklist (see Figure 2) to guide students' problem-solving processes. In Step 1, students were asked to detect the problem types on the basis of the problem structures they learned during problem-structure instruction. In Step 2, given the problem types, students were taught to use the corresponding conceptual model diagrams to organize the information or express mathematical relations in the equations. The first author developed WP story grammar prompting cards for each problem type (critical features for each of the four problem types are described in next section) to guide students' problem representations (see Figure 1 for WP story grammar prompt cards for each problem type). Students were allowed to use letters they preferred to represent unknown quantities. Step 3 required students to transform the diagrams to meaningful math equations. Students were taught to "peel off" boxes and labels in the conceptual model diagrams to make true mathematical equations. In Step 4, students learned to solve for the unknown quantities through equation manipulations. In addition, students were asked to provide complete answer to problems and check their answers to make sure they made sense.

*WP story grammar.* WP story grammar questions were designed to help understanding of deep WP story structure through the identification of three key elements in each problem type (e.g., part, part, and whole in PPW problem types). The prompt card for each problem type gives a definition of the problem type, followed by its conceptual model. Then a series of WP story grammar questions and prompts were generated in accordance with the model to help students correctly map information from the problems to the model diagrams (see Figure 1). As shown in Figure 1, the conceptual model diagrams emphasized the algebraic expression of the relations among three key elements of each problem type in an equation.

Specifically, a PPW problem described an additive relation between multiple parts and the whole (i.e., the parts make up the whole). It included problems such as combine (e.g., "Christine has 5 apples. John has 4 apples. How many apples do they have together?"), change-join (e.g., "Christine had 5 apples. John gave her 4 more apples. How many apples does Christine have now?"), and change-separate (e.g., "Christine had 9 apples. Then she gave away 4 apples. How many apples does she have now?") (Van de Walle, 2004). The placement of the unknown could be on the part or on the whole (see eight variations of PPW problems in Table 2a). An AC problem compared two quantities and involved a compare sentence that described one quantity as "more" (AC-more) or "less" (AC-less) than the other quantity (e.g., "Christine has 9 apples. She has 5 more apples than John. How many apples does John have?" or "Christine has 9 apples. John has 4 less apples than Christine. How many apples does John have?"). The placement of the unknown could be on the *big*, *small*, or *difference* quantity (see six variations of AC problems in Table 2a).

An EG problem described a number of equal sets or units. The placement of the unknown could be on the *unit rate* (the number of items in each unit or unit price), the *number of units* or sets, or the *product* (see three variations of EG problems in Table 2b). An MC problem compared two quantities and involved a compare sentence that described one quantity as a multiple or part of the other quantity. The placement of unknown could be on the *compared* set, the *referent* set, or the *multiplier* (i.e., multiple or part; see three variations of MC problems in Table 2b). (It should be noted that the MC problems in Table 2b

**Figure 1**
**Word Problem (WP) Story Grammar Prompt Card for PPW, AC, EG, and MC Problem Types**




include only those with multiple but not part relations, such as "2/3," because participants did not know operations with fractions during the intervention of this study.)

Overall, the instruction was delivered through explicit strategy explanation and modeling, dynamic teacher-student interaction, guided practice, performance monitoring with corrective feedback, and

**Figure 2**
**DOTS Checklist**

- Detect the problem type.
- Organize the information using WP conceptual model (e.g., Part + Part = Whole) diagrams.
- Transform the diagram into a meaningful math equation.
- Solve for the unknown quantity or variable in the equation and check your answer.

Note: WP = word problem.

independent practice. During independent practice, students were provided with a six-item independent worksheet to solve one of the two types (i.e., PPW or AC, or EG or MC) they had just learned. It should be noted that the conceptual model diagrams were provided on all modeling, guided, or independent practice worksheets during the intervention. However, they were gradually phased out on the worksheets when students worked on solving mixed WP types, and they were not provided during probe conditions.

# Results

## Effect on Arithmetic WP Solving

Figure 3 (for addition and subtraction WP solving) and Figure 4 (for multiplication and division WP solving) present two cohort participants' WP-solving performance during the baseline, intervention, postintervention, and generalization conditions. It should be noted that Kate completed only the PPW problem type of instruction because of the end of the school semester. As such, her performance data on the criterion tests were based only on one problem type (i.e., PPW), a total of 8 rather than 14 items.

## Addition and Subtraction WP Solving

*Baseline performance.* During the baseline condition, average performance across three participants on the criterion tests was 21% (median = 29%) correct for Deb, 28% (median = 21%) correct for Rita, and 28% (median = 21%) correct for Kate. Across the three participants, there were some variations in baseline assessment but no consistent pattern of either an increase or a decrease in trend. The low and stable baseline performance across the three participants indicated the need and set the stage for intervention.

*Intervention effect.* During the PPW problem-solving instruction phase, all three participants reached 100% correct in solving PPW problems during the second independent work probe. Deb scored 100% across two consecutive sessions, whereas both Rita and Kate missed one problem out of six (83% correct) on the first independent work probe.

During the probe condition, the first two participants (i.e., Deb and Rita) had to solve both problem types, as they did during the baseline condition. Deb scored 64% correct (an increase of 43% from baseline), and Rita scored 71% correct (an increase of 43% from baseline). This gain was primarily from the improvement in solving PPW problems, because analyses of students' work on the criterion tests (including both PPW and AC problems) indicated that their performance on "not yet trained" AC problems remained about the same as their baseline performance (compared with baseline, Deb solved one more AC problem, whereas Rita solved one fewer AC problem). The results indicated that the two participants improved their performance on trained (PPW) problems only but not on untrained (i.e., AC) problems. Because Kate's data collection was only on PPW problems, she performed at an average of 92% correct in solving PPW problems across two postintervention tests, indicating an increase of 63% from her baseline performance.

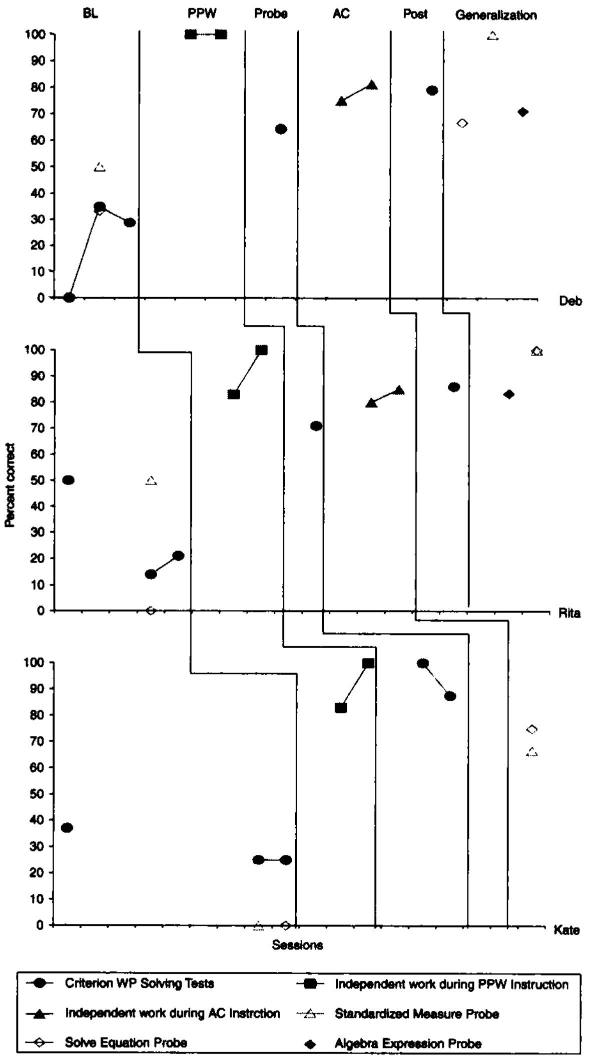
During the AC problem-solving instruction phase, the two participants who received instruction in AC scored an average of 78% and 83% correct. It should be noted that because of the approaching end of the school semester, there was not sufficient time left for AC problem-solving instruction (only three sessions for Deb and two sessions for Rita rather than four sessions, as planned, for one problem type).

Following one or two sessions on solving mixed WPs, including both PPW and AC types, Deb's performance on the postintervention criterion test was 79% correct, indicating 15% further increase from the post-PPW instruction probe or a 58% increase from baseline. In contrast, Rita's performance on the postintervention criterion test was 86% correct, indicating 15% further improvement from the post-PPW instruction probe or a 58% increase from baseline.

## Multiplication and Division WP Solving

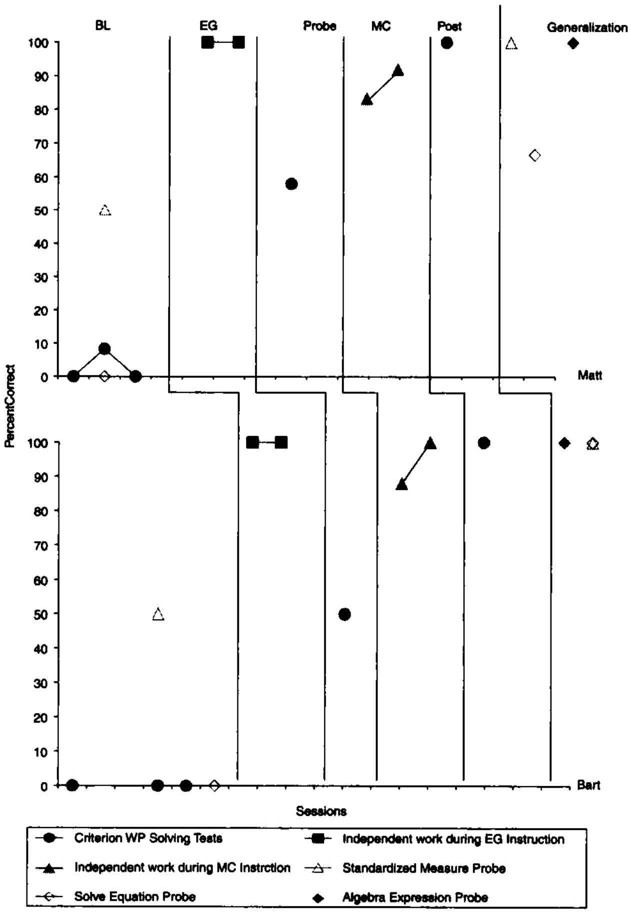
*Baseline performance.* During the baseline condition, average performance across the two participants

**Figure 3**
**Percentage Correct for Word Problem (WP) Solving During the Baseline, Intervention, Postintervention, and Generalization Conditions for Three Participants Who Received Strategy Instruction on Addition and Subtraction**




Note: AC = additive compare; BL = baseline; PPW = part-part-whole.

**Figure 4**
**Percentage Correct for Word Problem (WP) Solving During the Baseline, Intervention, Postintervention, and Generalization Conditions for Two Participants Who Received Strategy Instruction on Multiplication and Division**




Note: BL = baseline; EG = equal group; MC = multiplicative compare.

was 3% (median = 0) correct for Matt and 0% (median = 0) correct for Bart. The low and stable baseline performance across the two participants indicated the need and set the stage for intervention.

*Intervention effect.* During the EG problem-solving instruction phase, both participants reached 100% correct in solving EG problems, indicating mastery of this problem type. Following EG instruction, Matt scored 58% correct and Bart scored 50% correct on the criterion test involving both EG and AC types. On the basis of analyses of students' work on the criterion tests, both participants improved by 100% in solving trained EG problems and remained the same as their baseline performance on not-yet-trained MC problems (Matt solved one MC problem item, and Bart solved zero MC problems).

During the MC problem-solving instruction phase, Matt scored an average of 89% correct, and Bart scored an average of 94% correct. Again, because of the approaching end of the school semester, only two sessions of MC instruction were delivered to each of the two participants. Following one or two sessions on solving mixed WPs, including both EG and MC types, Matt's performance on the postintervention criterion test was 100% correct, indicating a 42% further increase from the post-EG instruction probe, or a 97% increase from baseline. Bart's performance on the postintervention criterion test also reached 100% correct, indicating 50% further improvement from the post-PPW instruction probe, or a 100% increase from baseline.

In addition, students were assessed on the standardized measure derived from the KeyMath-R/NU. Because three students received instruction on addition and subtraction only and two students received instruction on multiplication and division only, data on the standardized measure probe displayed in Figure 3 represent students' performance on four addition and subtraction items, and data displayed in Figure 4 represent students' performance on four multiplication and division items. As shown in Figure 3, both Deb and Rita improved from 50% to 100% correct from baseline to postintervention. Kate improved from 0% to 75% from baseline to postintervention. As displayed in Figure 4, both Matt and Bart increased their performance from 50% to 100% from baseline to postintervention.

As for strategy use, on the basis of the examination of participants' postintervention performance on each WP-solving probe, we found that all five participants used the conceptual model–based diagrams they had learned to express mathematical relations in the equations before providing answers to the problems.

### Effect on Prealgebra Concepts and Skills

On the solve equations probe, students were tested both before and after the intervention. As shown in Figure 3, Deb improved from 33% to 67% correct, Rita from 0% to 100% correct, and Kate from 0% to 67% correct from baseline to postintervention. As shown in Figure 4, Matt improved from 0% to 67% correct and Bart from 0% to 100% correct.

For the algebra expression probe, the participants of this study had no knowledge of what they were asked to do and made no attempts. After the intervention, they were retested on the algebra expression probe. As shown in Figure 3, Deb scored 71% correct and Rita scored 83% correct on the addition and subtraction algebra expression probe. No data were collected on Kate, because she did not complete the two problem types' instruction. For the second cohort, as shown in Figure 4, both Matt and Bart scored 100% correct on the multiplication and division algebra expression probe.

## Discussion

### Effects on Arithmetic WP Solving

Overall, the results indicated a gradual increase of students' performance from baseline to postintervention probes as the instruction on each problem type proceeded. Across three participants in the addition and subtraction instruction cohort and two participants in the multiplication and division instruction cohort, it is clear that each baseline changed only when the intervention was introduced. As such, we can attribute the effects to the intervention rather than other factors (Kazdin, 1982; Richards, Taylor, Ramasamy, & Richards, 1999). There seems to be a functional relationship between the intervention and improvement in participants' performance on the criterion tests. Across all five participants' data paths, there is no overlapping between baseline and postintervention performance. This is evidence of a strong treatment effect (percentage of nonoverlapping data = 100%; Scruggs & Mastropieri, 1998). In addition to evident internal validity, external validity was apparent because the effects were replicated across problem types (PPW to AC and EG to MC) and five individuals. Furthermore, all participants improved their performance on a measure derived from KeyMath-R/NU (Connolly, 1998), which further enhanced the integrity of the intervention effects.

### Effects on Prealgebra Concept and Skills

The results indicate that the intervention enhanced students' knowledge and skills in expressing mathematical ideas in equations and solving for unknown quantities in equations. As shown in Figure 1, the conceptual model–based WP story grammar facilitates the representation of mathematical relations in equations rather than teaching rules for decision making on the choice of operation for solution. Once students mapped information from problems to the conceptual model diagrams or algebraic equations, they were able to solve the problems by finding the values of the unknown quantities in the equations.

Traditionally, algebra instruction is postponed until adolescence because of historical reasons, including assumptions about developmental readiness and documented difficulties students encounter with algebra

(Carraher, Schliemann, Brizuela, & Earnest, 2006). Instead of viewing algebra as an extension of students' arithmetic experiences, a large number of adolescents do not connect algebraic concepts with previously learned ideas (Chappell & Strutchens, 2001). Chappell and Strutchens (2001) suggested two reasons why students fail to connect algebraic ideas to previous learning: (a) a lack of exposure to algebraic ideas and thinking and (b) students' tendency to learn algebra as mere symbol manipulation.

A growing consensus has emerged on the necessity to provide students the opportunity to engage in algebraic reasoning earlier in their education (Carpenter, Levi, Berman, & Pligge, 2005). The results of this study support the notion that algebraic reasoning can be integrated in arithmetic WP-solving instruction with students with MD and those at risk for math failure. The conceptual model–based representations promote algebraic reasoning in arithmetic WP solving and enhance algebra concept and skill acquisition.

## Limitations and Implications for Future Research

One limitation of this study is that a mastery-learning paradigm (Bloom, 1976) was not used to allow each student to reach mastery before moving onto the next phase or condition. Because of limited school days available for the study, it was not possible to allow each student to reach mastery. As shown in Figures 3 and 4, most participants did not reach 100% during the AC or MC instruction phase. Most participants received only about two sessions of instruction on the second problem type (i.e., AC or MC) before moving on to mixed review and phasing out the diagrams. The third participant in the addition and subtraction cohort did not complete instruction on the second problem type. As such, replication across problem types was not established for this participant. In addition, maintenance was not assessed, because of the ending of the school semester. It is important for future research to teach to mastery and assess students' retention of learned skills over time. In addition, it is important for future research to extend this preliminary study to involve problems with multiple steps and irrelevant information.

In addition, because of the limited number of students who consented to the study and the fact that those who did consent were not willing to take numerous tests, alternate-forms reliability for the criterion test was not established. However, because these alternate forms were generated by a computer program on the basis of predefined specifications, and the sequence with which students took alternate forms was randomized across the participants, the lack of correlation configuration should not confound the findings of this study.

## Implications for Practice

Observation of students' WP solving during the baseline condition indicated an immature impulse to grab numbers found in the problems and apply operations to produce answers for the solution. It is clear that these students bypassed conceptual model–based representations and moved "directly to mathematical expression on the basis of syntactical, surface clues" (Greer, 1992, p. 285). As summarized in Greer (1992), on the basis of findings from researchers such as Sowder (1988), the strategies students could possibly use include the following:

> Look at the numbers; they will tell you which operations to use.
> Try all the operations and choose the most reasonable answer.
> Look for key words or phrases to tell which operation to use. (p. 285)

To some degree, it was like gambling. That is, if an answer resulting from one operation did not work (e.g., did not result in a "good-sized" number or a "good-looking" integer), the students would try different operations. In short, this type of problem solving was not rooted in conceptual understating of problem structure or schemata, and the solution plans were not driven by conceptual models of problem situations (Jonassen, 2003).

Conceptual model–based problem solving emphasizes a conceptual understanding of problem structure. Key elements in the model (e.g., unit rate, number of units, and total or product in the EG problem type) could prompt the self-generation of WP story grammar questions to guide problem representation. Each WP story grammar question is directly related to each element in the conceptual model. On the basis of observation of participants' work following the intervention, students with or at risk for MD were able to articulate or describe key elements in the conceptual model of each problem type. Furthermore, some students were able to create their own WPs for each type following the intervention. WP story grammar questions helped students focus on relevant or important information in the problems and express mathematical relations in the conceptual model–based diagrams that led directly to the solutions. One implication for practice is that students with or at risk for MD were able to engage in algebraic reasoning, which involves the symbolic representation of mathematical ideas in equations.

When working on problem representation and solving for unknown quantities or variables, it is important to pay particular attention to the equal sign and unknown quantities or variables in algebraic equations. The symbolic expression of mathematical relations in equations may be a brand-new experience for some students. It was not surprising to see students exchanging the positions of terms in the equation as they wished, without worrying about "destroying" the equation or "tipping off the balance of the scale." It was also interesting to see students placing the bigger number (of two numerical values) found in a problem on the right side of the equation by itself (as the whole), without considering that the unknown quantity in the problem could be the biggest number. For instance, students had extreme difficulty in understanding and representing problems such as "Maya has 12 video games. Maya says she has 33 fewer video games than Naomi. How many video games does Naomi have?" One student was brave enough to say that it was a wrong problem: "How come Maya has 33 fewer games, since she only has 12 games?" Students had difficulty understanding that the unknown quantity (i.e., the number of games that Naomi had, 45) was bigger than the two numbers provided in the problem (i.e., 12 and 33). In short, it is important to prompt students to construct representations on the basis of a conceptual understating of the situations (not merely comparing the numbers "detached" from the problem contexts). The WP story grammar questions serve to guide this process.

## Conclusion

The four basic problem types (PPW, AC, EG, and MC) included in this study represent "the most common form of problem solving" (Jonassen, 2003, p. 267) in elementary school mathematics curricula. Learning to solve variations of these WPs is the basis for solving more complex, real-world problems (Van de Walle, 2004). Given the generalizable conceptual models of the four problem types as presented in this study (see Figure 1), a range of arithmetic WPs involving four basic operations can be represented and modeled. In addition, conceptual model–based problem solving (with the assistance of WP story grammar in representation) emphasizes symbolic or algebraic expressions of mathematical relations in equations and may facilitate a smoother transition from elementary to higher level mathematics learning. It is well documented that U.S. students begin to experience difficulty and to dislike mathematics after fourth grade, when learning becomes more abstract or symbolic and involves more algebraic thinking (Cai et al., 2004). As suggested by Booth (1988), "students' difficulties with algebra may result from the limited ways that they are taught about arithmetic and elementary mathematics" (Carraher et al., 2006, p. 92). It is imperative to integrate algebraic reasoning in elementary mathematics as endorsed in the NCTM (2000) standards and by the RAND Mathematics Study Panel (2003). With the inclusion movement and the passage of the No Child Left Behind Act (U.S. Department of Education, 2001) and the Individuals With Disabilities Education Improvement Act (U.S. Department of Education, 2004), students with disabilities or difficulties deserve no less.

Although story grammar has been substantially researched in reading comprehension (Boulineau et al., 2004), WP story grammar has not been explored in mathematics WP understanding and solving. Obviously, "it is readily apparent that comprehending verbal math problems involves processes different from those involved in comprehending other types of discourse, such as stories" (Bilsky, Blachman, Chi, Mui, & Winter, 1986, p. 109). Rather than focusing on textual analysis of story content, as emphasized in the story grammar in reading comprehension, the WP story grammar proposed in this study emphasizes analysis of deep problem structure and a "higher order units" mapping of key elements of problem schemata in conceptual models (Bilsky et al., 1986, p. 111). The generalizable conceptual model–based diagram presented in each WP story grammar prompting card serves to prompt learners to identify WP story elements and as a visual-spatial display for representing key elements in each of the four problem types. Furthermore, the algebraic expression of mathematical relations in the diagram or equation directly links problem representation to solution. The conceptual model–based WP story grammar that promotes the algebraic expression of mathematical relations in arithmetic WP solving may represent an innovative bridge between arithmetic and algebra learning.

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